PREDICTION OF LIFTING SURFACE FLUTTER AT SUPERSONIC SPEEDS†

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Summary—Accumulated evidence indicates that, for many low-aspect-ratio lifting surfaces and for mission profiles typical of many present-day aircraft, flutter may be a serious design problem at supersonic as well as at transonic speeds. Following a discussion of the effects of aerodynamic heating, measured flutter speeds and frequencies are presented for a related series of uniform, cantilevered rectangular wings at Mach numbers between 1.5 and 5.0. The remainder of the paper is devoted to various attempts at rational theoretical prediction of the experimentally determined eigenvalues. It is found that a two-degree-of-freedom representation based on free vibration modes of a uniform beam-rod is suitable for establishing equations of motion, but that the aerodynamic derivatives must be properly chosen for each particular speed range. One significant conclusion is that basic supersonic flutter theory has now received the same degree of confirmation that has long existed for straight wings of large span in incompressible flow.

SYMBOLS

a	Ambient speed of sound in air
b	Reference semichord of lifting surface (constant for the rectangular wings analyzed below)
$c = zb$	Chordlength of lifting surface
h_0	Amplitude of (positive downward) bending oscillation of elastic axis at wingtip
I_α	Mass moment of inertia in pitch about elastic axis, per unit span, at reference station on lifting surface
$(JG)_{REF}$	Torsional rigidity at reference chordwise cross section of lifting surface

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m	Mass per unit spanwise distance at reference station on lifting surface
$M = V/a$	Flight Mach number
$q = \frac{1}{2}pV^2$	Flight dynamic pressure
$V_d = \sqrt{I_\alpha/mb^2}$	Radius-of-gyration parameter
t	Maximum thickness of airfoil section at reference station on lifting surface
V	Flight speed or test-section flow speed in wind tunnel
V_f	Lowest speed for neutral dynamic aeroelastic stability (flutter) of lifting surface
w_1	Piston velocity
α_0	Amplitude of (positive leading-edge upward) torsional oscillation of wingtip
γ	Ratio of specific heats in air
$\mu = \dfrac{m}{4pb^2}$	Mass ratio
ϱ	Ambient density in air
$Ø_0$	Phase angle by which wingtip bending oscillation leads torsional oscillation
w	Circular frequency of simple harmonic motion
w_f	Circular frequency for neutral dynamic aeroelastic stability (flutter) of lifting surface
w_h	Frequency of fundamental mode of coupled bending vibration of wing
w_α	Frequency of fundamental mode of coupled torsional vibration of wing
w_{h_e}, w_{α_e}	Effective frequencies of uncoupled modes, calculated as described under Presentation of Data
O(...)	Identifies a quantity of the same or smaller order of magnitude than (...)

INTRODUCTION

IT is not difficult in the year 1960 to make a strong case for the need of accurate theory capable of predicting lifting surface flutter throughout the supersonic flight regime. Up to approximately 1945 the structural rigidity required to meet strength or static load criteria was generally more than adequate to prevent dynamic aeroelastic instability on fixed wings and tails (although the same could not be said about the trailing-edge controls and tabs of that era). But subsequent progress in aerodynamics and propulsion, coupled with the reduced thickness ratios permitted by improved materials and techniques of structural analysis, produced a steady increase in the ratio of flight dynamic pressure to stiffness to the point where satisfactory strength no longer assured freedom from flutter. Flutter prevention therefore became an important design consideration; new criteria and procedures were necessary.

The requisite information has gradually been supplied from a combination of analytical and experimental research. For transonic speeds, emphasis had to be placed on the reduced-scale model approach. A suitable theory was lacking at the same time that this range was recognized

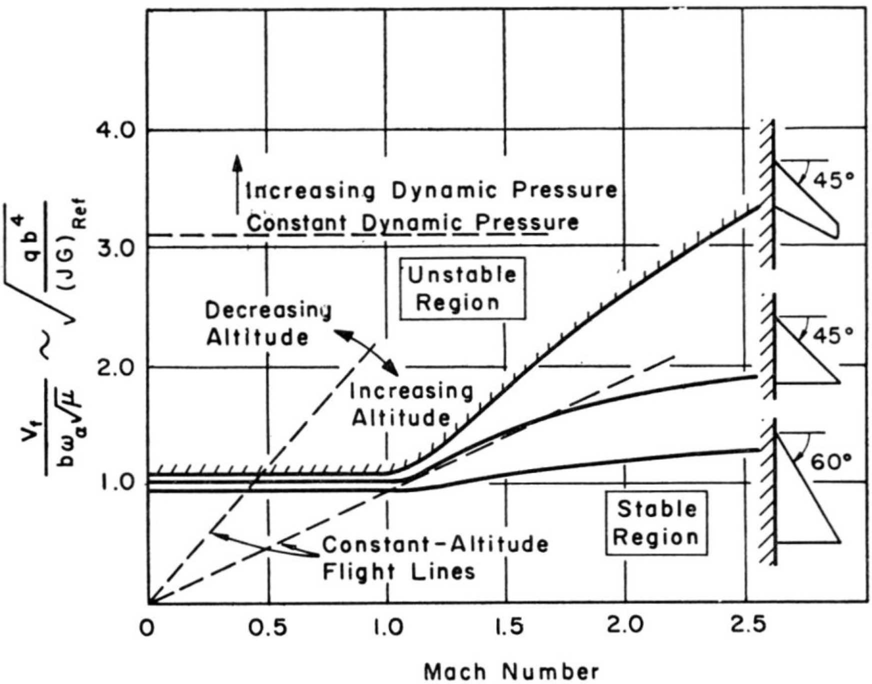

FIG. 1. Experimentally determined flutter stability boundaries, on a plot of dimensionless velocity index $V_f/bw_\alpha\sqrt{\mu}$ vs. Mach number, for cantilever model wings of three different planforms. Constant altitude flight is represented by a straight line through the origin. (Adapted from Garrick[1].)

as being critical from the standpoint of flight safety. Model tests, supplemented by semi-empirical analyses, ultimately supplied the data to establish the flutter boundaries, the margins of safety, the changes frequently needed to prevent instability and, in some cases, the optimum aircraft design from considerations of flutter. Extensive investigations had to be conducted on certain transonic configurations, such as fixed surfaces of very low thickness ratio, T-tails, mass-unbalanced control surfaces, wings with external stores and all-movable controls.

To illustrate current trends, the stability boundaries of Fig. 1 have been adapted from Garrick[1]. These curves are typical of what can be constructed from the large quantities of data now available near and above Mach number unity (e.g. Lauten and Burgess[2]). They show, for instance, that the particular swept wing chosen here is critical at transonic speeds; if no flutter occurs up to $M \simeq 1$, it can proceed safely far into the supersonic range, even at constant altitude.

Throughout the present paper we use the velocity index $V_f/b\omega_a\sqrt{\mu}$ as a measure of the tendency to flutter. The changes in this index near $M = 1$ can be attributed to the variations in lift-curve slope and aerodynamic center as a wing accelerates from subsonic to supersonic flight. The rapid rise of the velocity index or the supersonic alleviation is not so great, however, as originally expected from zero thickness, linearized aerodynamic theory. The unconservatism of linear theory, which was pointed out by Ashley and Zartarian[3], results primarily from a forward shift of aerodynamic center due to profile shape and thickness. In Fig. 1, the stability boundaries for the two delta wings tend toward constant values of velocity index at the higher M, this tendency being more pronounced for the higher leading-edge sweep. It can be speculated that this behaviour results from the smaller variation of aerodynamic parameters with M as the aspect ratio is decreased. In any event, the immediate transonic region is not necessarily the most dangerous for the deltas. Thus, the relationship between the constant-altitude line and the boundary for the 45°-planform indicates a critical range extending roughly from $M = 1$ to 1.5. If this wing is designed to fly at appreciably greater q supersonically than transonically, the first encounter with flutter may be distinctly supersonic. The same can be said for the 60°-delta, which displays an even wider range of M where instability could be met.

Although the foregoing examples are not all-inclusive, the data do suggest that the supersonic regime may be a critical one for many low-aspect-ratio surfaces. This statement is substantiated by the observation that dangerous high-q regions at low altitudes may be avoided through deliberately specified or accepted speed placards. Clearly the means must be at hand for preventing flutter at supersonic speeds and, possibly for some unique designs, at hypersonic speeds.

Flutter models may be employed to determine stability information for specific configurations on a direct-analog basis, to develop trend data of the sort presented in Fig. 1, or to assist in establishing and improving the accuracy of procedures for theoretical prediction. It was with the latter two objectives in mind that the measurements discussed in this paper were carried out. These were conducted by the Aeroelastic and Structures Research Laboratory, M.I.T., at the request of Wright Air

Development Division, USAF, (1) to provide systematic data on a series of similar models in the range of $1.5 \leqslant M \leqslant 5.0$, and (2) evaluate the accuracy of the supersonic analytical prediction technique known as "piston theory". This was a scheme adapted by M.I.T. investigators from earlier theoretical formulations of Hayes[4] and Lighthill[5]. Such information is required at a sufficiently early date to advance the "state of the art" and to underwrite the design of advanced flight vehicles.

AEROTHERMOELASTIC EFFECTS

Before the experimental results are set down and compared with their predicted counterparts, a few words are in order about the influence of aerodynamic heating on supersonic flutter. At high speeds the destabilizing effects of steady-state and transient heating on rigidities will require consideration not only because of their influences on frequencies but also on frequency ratios. Garrick[1] describes results of a NACA hot jet experiment on a solid bending-torsion model. This model fluttered for a short interval primarily on account of loss in rigidity due to transient thermal stresses and the associated changes in frequency ratios and frequencies. It was entirely stable, however, when injected into a similar cold jet (see also Runyan and Jones[6]).

The potential impact of aerodynamic heating on dynamic and static aeroelasticity was early recognized and stimulated extensive research. References (7,8,9) are a small sample of U.S. documents reporting results of investigations. Dryden and Duberg[9] describe an interesting and complicated aerothermoelastic phenomenon involving significant amounts of chordwise deformations. This "flag-waving flutter" occurred during hot jet tests on a multiweb wing. Chordwise deformations will very likely require much more attention in future designs.

At the present time, the general procedure in aeroelasticity is to separate the aerothermoelastic problem into two phases—the aerothermal problem and the aeroelastic one. Bisplinghoff and Dugundji[10] show that such a procedure implies two assumptions, which are (1) small coupling between heat transfer and elastic deformation, and (2) small coupling between static and dynamic aerothermoelastic effects. However, these M.I.T. investigators also point out that dynamic coupling between aerothermal and aeroelastic aspects is unlikely, since thermal transient characteristic times are generally long when compared to periods of structural vibration important to flutter.

Many investigators have proved that exact aerothermoelastic simulation is not feasible except for a 1:1 model or replica. It should be noted that, because of aerodynamic heating effects on moduli and changes in rigidities due to thermal stresses, the determination of the experimental time scale must consider both aircraft and model histories. That is, aerothermoelastic phenomena are a function of aircraft flight path and maneuvers. While the rocket-model approach may appear to give more realistic combinations of dynamic pressure and temperature than the "non-heated" wind tunnel, the additional rocket propulsion and aerodynamic performance similarity parameters required from flight path considerations or the altitude-speed limit boundary may not permit complete aerothermoelastic simulation. Nevertheless, the rocket-model technique does present a means for obtaining high temperatures and high dynamic pressures. Use of this experimental tool will likely be expanded to obtain data for developing and improving both experimental and analytical approaches in resolving flight vehicle aerothermoelastic problems. Moreover, limitations on available stagnation temperature render the continuous wind-tunnel even more deficient in this respect.

Calligeros and Dugundji[11] evaluate similarity requirements up to 1000°F or about $M = 3.5$. They point out that the primary conflict is between the Mach number, Reynolds number and pressure-structural modulus ratio conditions. Of lesser importance are conflicts of characteristic time, radiation and Frounde number effects. They also observe that exact similitude is possible only for a 1:1 scale ratio. Three possible approaches are discussed by the authors in detail:

(1) to use different materials and test media (gas) for the model;

(2) to investigate special conditions by considering plate-like behavior; and

(3) to relax one of the major conflicting requirements.

In the latter case, the Reynolds number condition is shown to be a redefinition of the time scale. (See also Garrick[1] for a similar discussion on time sequence dissimilarity.)

In view of the similitude conflicts briefly mentioned above, it will be necessary to employ restricted-purpose models and, in many cases, to separate the aerothermal and aeroelastic phases of the problem. Nevertheless, as in the past, these models will be indispensable for investigating special aspects of the various phases of the aerothermoelastic problem area. Obviously such test results can be employed to provide data for the development of theoretical approaches and to evaluate and improve the accuracy of analytical prediction methods. In some cases, the model may be used as an approximate analog of the full-scale vehicle. For the "quasi-steady thermodynamic approach", the effective stiffnesses, including increases or reductions due to thermal stress, are estimated from analyses and possibly test data and are specified as functions of time.

A series of models is then constructed and tested to evaluate the more critical "time-frozen" conditions.

Considerable research is required to develop and to validate similarity procedures in aerothermoelasticity, especially with reference to relaxation of conflicting similarity requirements. Rocket model tests will provide significant information on combined high heating and aerodynamic pressure effects. However, here again conflicts in similarity will likely require tests with different geometric scale ratios. It is probable that exact or nearly exact simulation by ground facilities and reduced scale models will not be possible. Thus, an accurate validation of the various step-by-step phases of the separation technique will be required. The final model tests might involve some limited combined environmental tests either in a heated wind tunnel or by means of rockets. A decrease in the extent of exact similarity will of course emphasize the importance and the need for the flight flutter test. Here again the question of the influence of flight path must be evaluated. In addition to the effort needed in the area of aerothermoelastic similitude requirements, considerable research and development are needed in connection with the design, construction and testing of both static and dynamic aerothermoelastic models.

The above-mentioned effects of airplane flight path or history on flutter characteristics have been investigated by California Institute of Technology for specific cases under United States Air Force Contract AF33 (616)–5767. The results, reported by Harder *et al.*[12], cover analytical studies of a hypothetical straight wing in the speed-altitude region $M = 3$ at sea level to $M = 5$ at 50,000 feet. One analog computation concerned a long flight at $M = 5$ and 50,000 feet followed by a reduction in speed to $M = 1.5$, an $M = 1.5$ dive to sea level, and an acceleration to $M = 3.0$. This maneuver was sufficient to lose a considerable "cold-wing" margin of safety. Flutter of the hypothetical wing resulted at sea level as $M = 3.0$ was reached. The loss in torsional rigidity was due to a reduction in elastic moduli caused by "hot soak" at high altitude and to detrimental thermal-stress effects from the low-altitude acceleration to $M = 3.0$. The influence of vibration amplitude on effective rigidities and flutter stability was not investigated.

The research discussed in the following sections concerns tests in an unheated tunnel and thus no aerothermal effects are included.

PRESENTATION OF DATA

Very little experimental information on flutter at distinctly supersonic speeds is to be found in the literature, particularly for $M = 3$ and above. An important exception is the NACA Research Memorandum by Runyan and Morgan(13), which shows a reasonable agreement between piston strip-theory calculations and measured stability-boundary locations at $M = 3$ and 7 for 11%-thick double-wedge and 4%-thick truncated-double-wedge wing models. Although these correlations for these limited

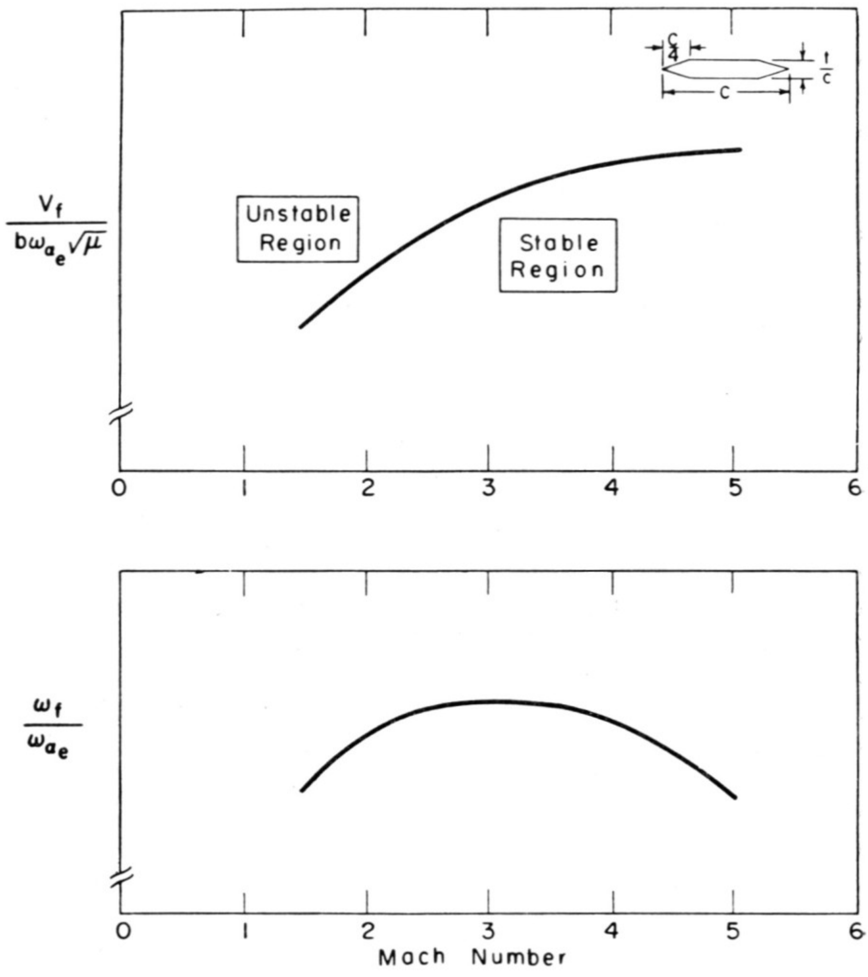

FIG. 2. Values of $V_f/bw_{\alpha_e}\sqrt{\mu}$ and w_f/w_{α_e} vs. Mach number measured on cantilever, half-span models of square planform and 4-%-thick, truncated double-wedge profile shape. Other model parameters typical of current practice.

cases suggest optimism regarding the theory, extended research involving systematic variations of Mach number and other wing properties still appears to be desirable.

Figures 2–4 summarize the result of the test program, which was designed to supply such systematic information regarding the effects of the parameters M, thickness ratio, t/c, and aspect ratio. The other dimensionless characteristics of model wings were held constant as nearly as possible, and are typical of current practice. Further details are reported by Martuccelli[14].

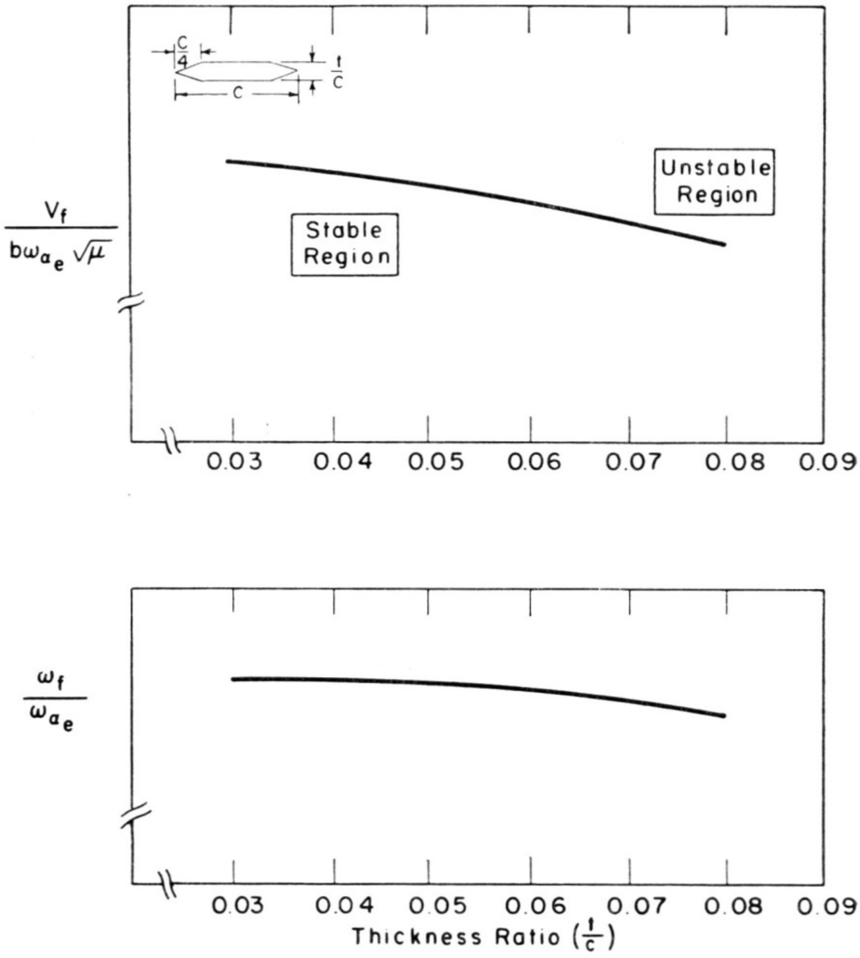

FIG. 3. Values of $V_f/bw_{\alpha_e}\sqrt{\mu}$ and w_f/w_{α_e} vs. thickness ratio t/c measured on cantilever, half-span models of square planform and truncated double-wedge profile shape at $M = 3.0$. Other model parameters typical of current practice.

The type of model construction employed was the metal spar balsa-wood profile arrangement used in other flutter research programs carried out at M.I.T.[15]. The technique is inexpensive and yields structures which display clearly defined beam-rod elastic characteristics and tend to flutter in the cantilever, first bending-first torsion mode. The balsa gives the desired aerodynamic shape, and airloads are transmitted by it to an aluminum spar, whose thickness and width can be varied to control the flexural and torsional rigidities of the model and whose chordwise location fixes the elastic axis. Lead weights are distributed fore and aft of the spar

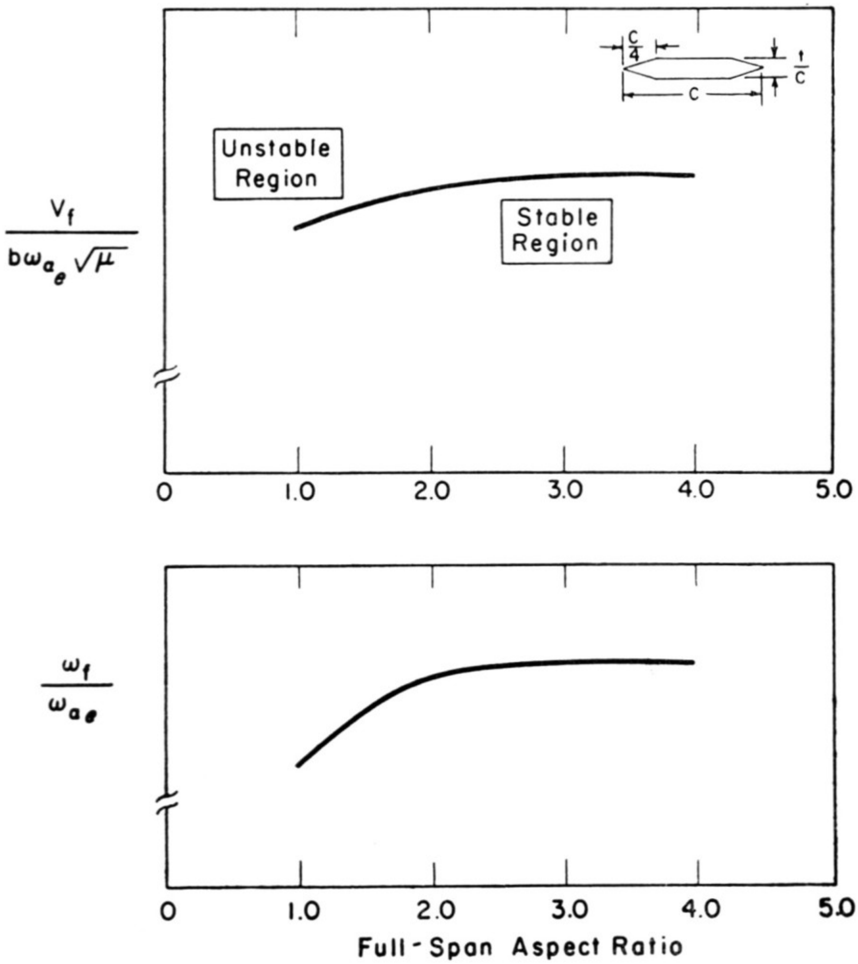

FIG. 4. Values of $V_f/bw_{\alpha_e}\sqrt{\mu}$ and w_f/w_{α_e} vs. full-span aspect ratio measured on rectangular, cantilever, half-span models with 4%-thick, truncated double-wedge profile shape at $M = 3.0$. Other model parameters typical of current practice.

in such a way as to give the desired mass, mass unbalance, and radius of gyration. Model instrumentation consists of two strain-gage bridges, which supply information on bending and torsional strains at the root. The gages are suitable for quantitative measurement of frequency and damping of the aeroelastic modes during zero-airspeed vibration tests and measurement of frequency during the flutter determinations.

The wind tunnel tests were performed in Tunnel E-1 of the Gas Dynamics Facility, USAF Arnold Engineering Development Center, Tullahoma, Tennessee. Tunnel E-1 is an intermittent-flow facility with a Mach number range of 1.5 to 5.0. The problem of the starting and stopping shock loads was avoided by injecting the model into the airstream after the starting shock had passed the test section and, when required, by retracting the model before stopping the tunnel. Flutter was obtained by increasing stagnation pressure (density-variation method) at fixed M.

Models incorporating a basic set of dimensionless parameters were tested at each half-Mach number in the operating range. These were half-span, wall-mounted, cantilevered wings with square planform (full-span aspect ratio 2.0 because of the aerodynamic and structural symmetry). Properties were uniform in the spanwise direction. The basic profile shape was that of a truncated, double wedge with a thickness ratio of$_4$ per cent.

The effect of Mach number on the flutter index $V_f/b\omega_{\alpha_e}\sqrt{\mu}$ and frequency ratio $\omega_f/\omega_{\alpha_e}$ for the basic models is shown in Fig. 2. Circular frequency ω_{α_e} is the effective frequency of the "uncoupled" fundamental torsion mode in vacuo, as calculated from cantilever free-vibration equations using the measured frequencies of the first two inertially coupled modes of the model. The influence of thickness ratio at $M = 3.0$ was determined by testing models similar to the basic configuration but with thickness ratios of 3, 4, 6 and 8 per cent. The results are shown in Fig. 3. Finally, the importance of three-dimensional flow at $M = 3.0$ was assessed by test results from four related similar basic models with $t/c = 4\%$ and full-span aspect ratios of 1, 2, 3 and 4 (Fig. 4).

At least two positive measurements of flutter eigenvalues were made on separate models at each Mach number except 2.5 and 3.5, including a total of 5 at $M = 3.0$ on the basic configuration. The curves in Figs. 2–4 were faired through these data points, fitting quadratic parabolas to them by means of a least-squares procedure. This process yielded a reasonable representation in all cases except the $\omega_f/\omega_{\alpha_e}$ curve of Fig. 4, which had to be drawn by eye centrally through the data. A careful analysis of all significant sources of experimental inaccuracy was carried out to determine the probable error of the measurements. This estimate was confirmed by placing a band of width equal to the experimental error around each of the curves; few individual points fall outside this band. The largest deviations occur in the values of frequency ratio $\omega_f/\omega_{\alpha_e}$ vs. M (Fig. 2), but they do not invalidate the predicted error.

COMPARISON BETWEEN THEORY AND EXPERIMENT

Assuming for the moment that the system has two degrees of freedom, one in bending and one in torsion, a dimensional analysis of the complex eigenvalue problem leads to the following conclusion: the dimensionless flutter speed $V_f/b\omega_{\alpha_e}$ and frequency ratio $\omega_f/\omega_{\alpha_e}$ for lifting surface with fixed planform geometry and affine profile shapes are functions of mass ratio μ, structural frequency ratio $\omega_{h_e}/\omega_{\alpha_e}$, Mach number M, thickness ratio t/c, radius-of-gyration parameter Γ_α, and the dimensionless chordwise locations of the elastic axis and center of gravity. This listing omits the influence of structural friction, which was very small for all models and, on theoretical grounds, would cause no more than a 1 per cent change in any measured speed or frequency. The last three qualities in the foregoing list were held within extremely close tolerances throughout the program. M and t/c were varied in a prescribed fashion. A word must be said, however, regarding the more significant variations in μ and $\omega_{h_e}/\omega_{\alpha_e}$ from one test to another.

Owing to the fact that ambient density in the wind tunnel becomes progressively lower as M is raised, the mass ratio μ at flutter increased by a factor exceeding 2 between the low and high ends of the speed scale. Since this behavior seems unavoidable in any such experiments, it is fortunate that a large amount of theoretical and experimental evidence has been accumulated (e.g. Ashley and Zartarian[(16)]) which shows, for a variety of wings and modes of instability in the ranges of aeronautical interest, that V_f is almost exactly proportional to $\sqrt{\mu}$. That is, if $V_f/b\omega_{\alpha_e}\sqrt{\mu}$ is employed to describe stability boundaries, the mass ratio can be dropped from the enumeration of parameters above. On a similar basis, ω_f turns out to be essentially independent of μ.

As for the frequency ratio $\omega_{h_e}/\omega_{\alpha_e}$, the exigencies of model construction caused it to deviate up to 30% from the nominal value desired, the higher ratios being associated with the lower Mach numbers. In the calculations to be presented, the mean $\omega_{h_e}/\omega_{\alpha_e}$ for all models tested at each M was inserted into the flutter equations corresponding to that M. Inasmuch as the eigenvalues are not particularly sensitive to moderate variations in this parameter about its nominal value, it is believed that this difficulty has negligible effect on the soundness of the conclusions reached below.

No effort is made here to reproduce details of flutter computation procedures or unsteady aerodynamic theory, these subjects being well covered in the literature to be cited. As a first, rather elementary attempt at correlation with the data in Figs. 2–4, the characteristic determinant was constructed by the Rayleigh–Ritz method, using generalized coordinates associated with the fundamental, uncoupled modes of flexural and torsional vibration of a uniform cantilever (cf. Eqs. (9–89) and (9–90) of[(17)]). Running lift and pitching moment were taken from second-order piston theory[(3)], without adjustment for induction or wingtip losses but taking account of the actual profile shape and thickness. The neutrally stable solutions are plotted in Figs. 5 through 7. Curves obtained by

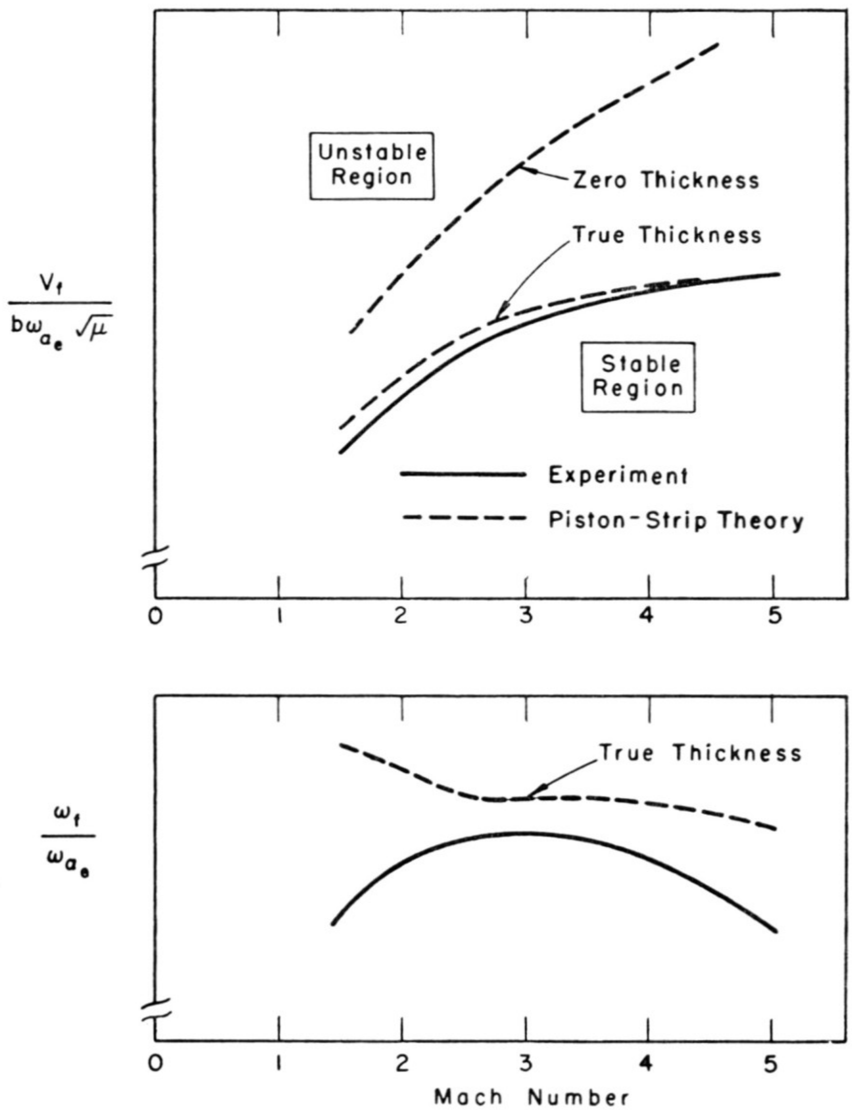

FIG. 5. Data of Fig. 2 compared with stability boundaries calculated using two uncoupled modes of a uniform, cantilever beam-rod and aerodynamic coefficients from second-order piston theory.

setting $t/c = 0$ have been added to the velocity index plots of Figs. 5 and 7. The serious unconservatism of the zero-thickness predictions confirms the well-known[3] destabilizing influence of the forward shift in aerody-

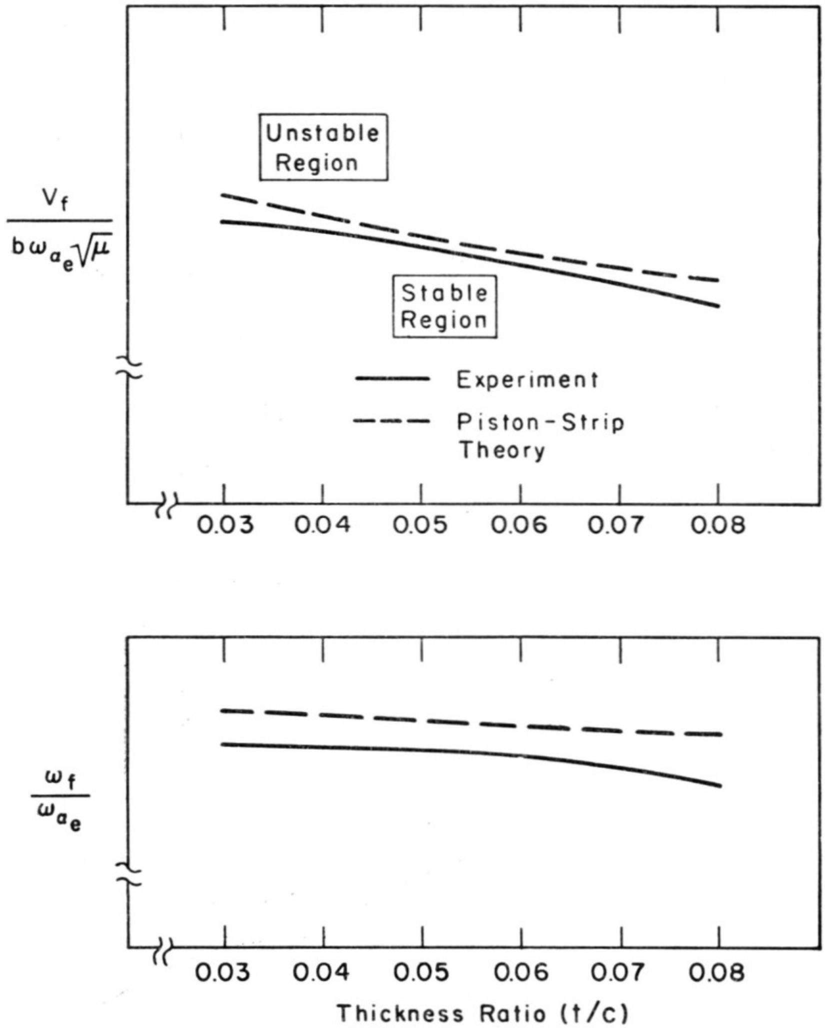

FIG. 6. Data of Fig. 3 compared with stability boundaries calculated using two uncoupled modes of a uniform cantilever beam-rod and aerodynamic coefficients from second-order piston theory.

namic center proportional to the parameter Mt/c. For straight wings of the general type represented here, the conclusion seems inescapable that wholly-linearized aerodynamic derivatives are unsatisfactory for estimating supersonic aeroelastic stability.

Despite a tendency to be slightly unconservative, piston theory predicts the location of the stability boundary of Fig. 5 to within the probable experimental error for $M \geqslant 3$. The same is not true of the calculated frequencies, although the apparent discrepancies between the $\omega_f/\omega_{\alpha_e}$

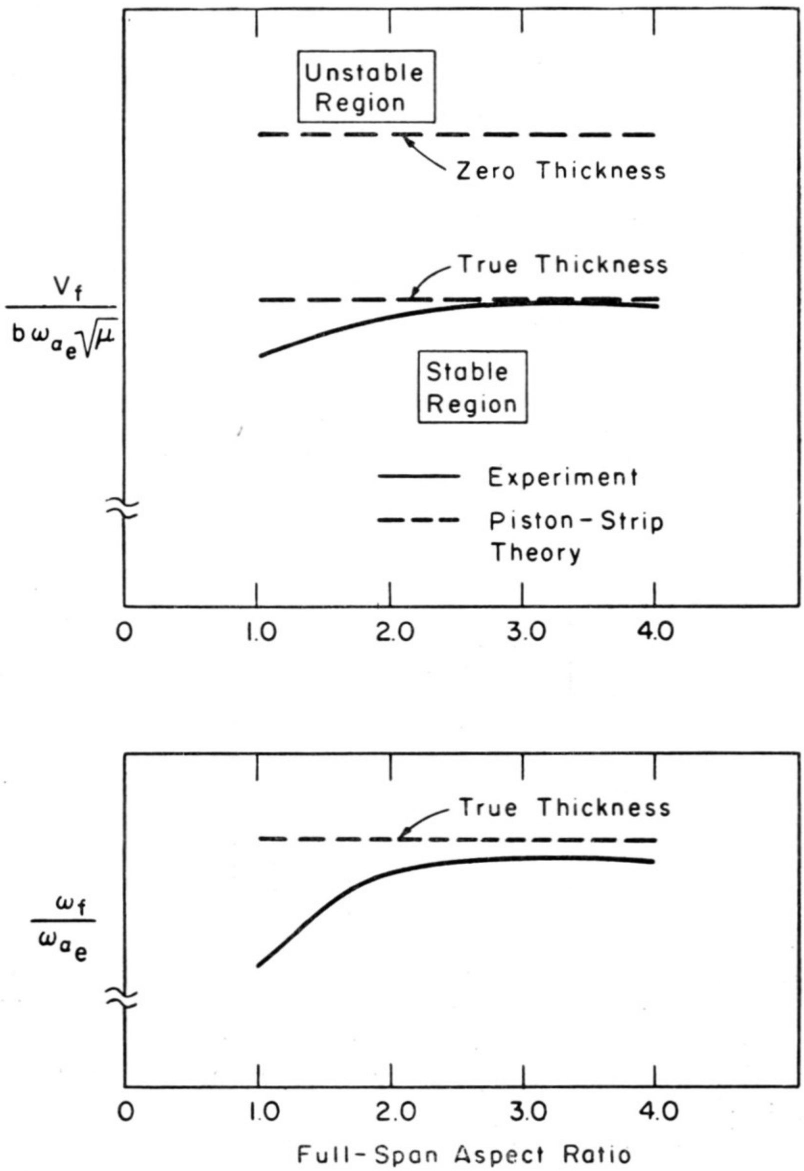

Fig. 7. Data of Fig. 3 compared with stability boundaries calculated using two uncoupled modes of a uniform cantilever beam-rod and aerodynamic coefficients from second-order piston theory.

curves are magnified by a scale expansion which is roughly twice that of the velocity-index scale. As might be expected, the greatest frequency deviation is observed at the lowest M, where the validity of the theory is seriously questionable.

It is a matter of common experience among aeroelasticians that frequency ω_f is predicted less accurately by a given theory than speed V_f. An even more sensitive index—but one that is rarely available—is the flutter mode shape. During the present program, high-speed motion pictures were taken of each test with the camera pointed very nearly along the elastic axis. It was afterward discovered that rough measurements of the bending and torsional amplitudes at the wingtip, and of the phase angle ϕ_0 by which the bending oscillation (positive downward) leads the torsion (positive leading-edge up), could be extracted from the films. In the table which follows, a representative sample is given of measured and predicted values of ϕ_0 and amplitude ratio h_0/b_{α_0}. The experimental errors for these two quantities are estimated at $\pm 10°$ and $\pm 20\%$, respectively. Nevertheless, the comparisons leave something to be desired, especially at the higher M. It would seem that more use should be made of careful determinations and correlations of experimental and analytical mode shape data for a critical evaluation of flutter theories.

TABLE 1

Representative mode-shape data

M	Experimental		Theoretical	
	h_0/ba_0	φ_0	h_0/ba_0	φ_0
1.5	1.51	27°	1.55	13.9°
2.0	1.28	13°	1.70	11.7°
3.0	1.59	14°	1.24	9.0°
4.0	1.50	9°	0.92	5.9°
5.0	1.77	30°	0.92	4.1°

In view of the reasonably satisfactory agreement on speed and frequency at $M = 3.0$ achieved for the basic model configuration, it is not surprising that Fig. 6 displays correctly estimated trends of these two quantities with varying thickness ratio. Regarding Fig. 7, the theoretical curves are horizontal straight lines because none of the dimensionless system parameters is changed when strip-type derivatives are employed and the aspect ratio is varied in such a way as to keep $\omega_{h_e}/\omega_{\alpha_e}$ constant. In actuality, the only significant influence of span comes about through the changing fraction of the total plan area that senses the presence of the wingtip, and only for aspect ratios less than 2 is this appreciable. At $M = 3.0$ a thin rectangular wing of aspect ratio 2 has 17.7% of its area so affected and can be considered, for flutter purposes, as an aggregate of two-dimensional airfoils. The reasons are discussed below.

When trying to improve a flutter theory, one must examine the approximate representations of both the structural and aerodynamic parts of the coupled system. Free-vibration tests of the models give no cause to suspect the assumption of beam-rod elastic behavior, but the use of only two degrees of freedom in the equations of motion may be questioned. To meet this criticism, parallel computations based on the nominal system parameters have been made which incorporate the following successive dynamical refinements: three degrees of freedom, with the second mode of uncoupled bending added; and an infinity of degrees of freedom, as obtained by means of the "exact" solution of the uniform beam-rod differential equations due to Goland[18], and Runyan and Watkins[19]. With subscripts "2", "3" and "∞" identifying the respective solutions, a typical set of flutter eigenvalues is set forth in Table 2. The aerodynamic derivatives are taken from second-order piston-strip theory in all cases. Since there is uncertainty about the third place after the decimal point in some of these computations, the two-degree-of-freedom approximation seems extraordinarily well justified. Care should be observed, however when generalizing any such finding to other, more complicated systems. In every case here, the mode which first became unstable is the one connected with torsional vibration at zero airspeed and involves an almost-pure admixture of fundamental bending and torsion. Throughout the multi-degree-of-freedom solutions, no hint of impending instability in other modes was found up to speeds much higher than V_f.

TABLE 2

Comparison between bending-torsion flutter speeds and frequencies calculated using two, three, and an infinite number of degrees of freedom in the dynamical system

M	$\frac{V_{f3}}{V_{f2}}$	$\frac{w_{f3}}{w_{f2}}$	$\frac{V_{f\infty}}{V_{f2}}$	$\frac{w_{f\infty}}{w_{f2}}$
1.5	1.009	1.002	1.002	1.002
3.0	1.004	1.001	1.000	1.001
5.0	0.999	1.001	0.998	1.001

Aerodynamic refinements can be undertaken with two objectives: to remove the restriction on piston theory to high values of M, which is clearly not met down to $M = 1.5$; and to allow for three-dimensionality of the flow. Landahl[20], Van Dyke[21] and others have provided airload expressions for airfoils oscillating at lower supersonic Mach numbers while retaining the all-important ingredient of nonlinearity. The first of these theories is somewhat easier to apply, and the results of intro-

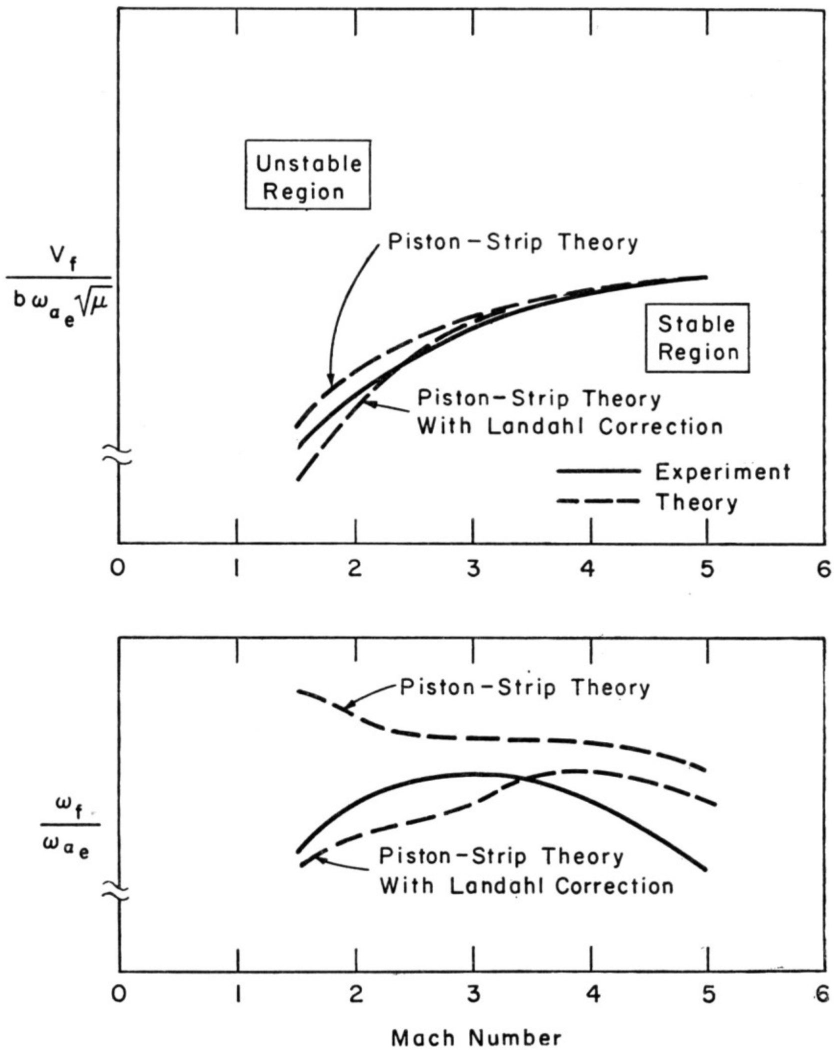

FIG. 8. Data of Fig. 2 compared with stability boundaries calculated by three-degree-of-freedom analysis using aerodynamic coefficients from second-order piston theory, with and without the correction terms from Landahl[20].

ducing it into the modal flutter calculations are presented in Fig. 8. Landahl criticizes second-order piston theory on the grounds that it includes effects $0(t^2/c^2)$ while neglecting those which are $0(t/cM^3)$, thus implying that $t/c \gg l/M^3$. For the 4%-thick profiles used in the present investigations, this would strictly call for $M > 4$, at least. The Landahl formulation[20] expands the velocity potential, pressure coefficient, etc., in increasing powers of t/c and l/M^2. His objections are quite well met by retaining only the piston terms plus those $0(t/cM^3)$, and this is what has been done.

The improved velocity-index curve of Fig. 8 can now be said to fall within experimental error when $M \geqslant 2$. Once again, the agreement on flutter frequency is less satisfactory. It is, nevertheless, much more uniform throughout the Mach-number range.

To combine thickness effects with three-dimensional flow in an entirely rational manner is not yet within the state of the art. Yet there is a possibility, when the eigenvalues are not excessively sensitive to the thickness parameter (cf. Fig. 6), of adjusting linearized theory on a semi-empirical basis. For instance, one may take the distributions of oscillatory pressure loading computed by means of supersonic aerodynamic influence coefficients[22,23] and supplement them by the term $\gamma(\gamma_{+1}) M^{dzt}/\mathrm{d}x \dfrac{W_1}{a_\infty}$ prior to integrating the generalized forces used in the flutter equations. The quantity $\mathrm{d}z_t/\mathrm{d}x$ represents the local chordwise slope of the semi-thickness distribution of the wing, and the term itself is easily deduced from piston theory. Alternatively and less rigorously, one can developed flutter curves from linearized computations, then adjust the ordinates by the ratio between corresponding curves obtained by two dimensional aerodynamics with and without the thickness effect.

The three-dimensional calculations without thickness, as expected, yield a very unconservative prediction of flutter speed, the variation with Mach number being not substantially different from the upper dashed curve on Fig. 5. After a rough thickness adjustment of the latter type discussed above, results are found which closely agree with the Landahl curve on Fig. 8 but which are no nearer than it is to the measured speeds. This lack of improvement at the lower end of the Mach number is due to the fortuitous accuracy of the two-dimensional estimates, and three-dimensional theory should certainly be used in other, more complicated cases. The success of the Landahl method at $M = 1.5$ and 2 can be attributed here to the mutual cancellation of two errors of opposite sign. On the one hand, the oscillatory loads drop off to zero as the wingtip is approached, but, on the other, what loads are present become more effective in producing flexure-torsion instability by virtue of the forward displacements of the sectional aerodynamic centers in the tip region.

CONCLUSIONS

For many classes of lifting surface and for mission profiles typical of the operation of many modern aircraft types, flutter may be a serious design problem at supersonic as well as at transonic speeds. The parabolically shaped curve of critical velocity V_f vs. Mach number in the supersonic regime, suggested for several years by theoretical studies on simple wings, appears to be well justified by the experimental evidence.

With regard to rational methods of flutter prediction in the general range $1.5 \leqslant M \leqslant 5.0$, it seems fair to conclude that they have now received the same sort of confirmation that was provided for the theory of subsonic, bending-torsion flutter of straight wings by a series of investigations in the late 1930's [24,25] are good examples from the British and American literature). As with all such statements concerning aeroelastic stability, this conclusion is a tentative one, based on tests with a single structural configuration in which several of the many important parameters had fixed values throughout. It must be emphasized that cases can be constructed where the eigenvalues are extremely sensitive to a certain parameter and where no theory can ever be successful. Such is the situation on an unswept wing when the frequency ratio w_h/w_α is small and the chordwise positions of center of gravity and aerodynamic center are close together.

In setting up flutter calculations, proper regard must be had both for the structural complexity and for the suitable choice of aerodynamic derivatives. Experience at lower flight speeds can serve as a guide in the first instance. On lifting surfaces of the general shape and aspect ratio dealt with here, what is acceptable aerodynamically depends on the Mach number. Unadjusted, second-order piston theory appears valid between about $M = 3$ and $M = 5$, and possibly to an upper limit fixed by the size of Mt/c, where entropy changes and other distinctly hypersonic phenomena prevent accurate prediction of the pressure distribution by piston theory. Proceeding downward from $M = 3$, a two-dimensional adjustment such as those proposed by Landahl[20] and Van Dyke[21] must first be introduced. Finally, there will be Mach numbers above the transonic range where only three-dimensional airloads, adjusted semi-empirically for thickness effect, can yield satisfactory predictions for the purposes of final design.

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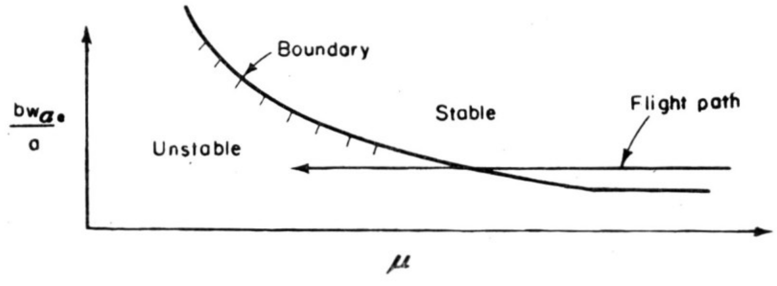
DISCUSSION

G. H. LEE: I should like to ask whether the author has any information on the variation of the subcritical damping with speed. In particular, is there any information on the comparison between theory and test?

In a case with which I am familiar, for an aeroplane flying at high subsonic speeds, there was a considerable discrepancy between the variation of damping with speed as determined by calculation, by wind-tunnel test and by flight test. In this case, the discrepancy was in the dangerous way, the calculations showing a gradual reduction in damping with speed, the wind tunnel tests a moderately rapid reduction, and the full-scale flight tests a very rapid reduction indeed; this resulted in the loss of an aeroplane. Information on this matter may therefore be of considerable importance.

H. ASHLEY and J. R. MARTUCCELLI: We made no measurements of variation of subcritical damping with speed and made no calculations either.

At the higher Mach numbers, where the flutter occurred at higher μ's, we were working in a region whose flight line and flutter boundary were roughly parallel.



At $M = 5.0$ we actually obtained a sustained limited amplitude flutter at the lowest possible μ which indicates that the boundary and flight path were roughly parallel.

J. WILLIAMS: Professor Ashley intimated that the onset of flutter was violent in the experiments. Was the flutter simply spontaneous and did "bumping" make any noticeable difference to the critical speed? Could Professor Ashley also comment on the effects of changes in wing incidence setting?

H. ASHLEY and J. R. MARTUCCELLI: Flutter was spontaneous in all cases. I presume that "bumping" means to give model a jolt so as to initiate flutter. This we never found necessary.

We did make two runs where we approached the flutter boundary very slowly, so as to try and save the model after flutter began. We were able to do this and were able to save the models. In both cases we tested the models again under the same tunnel conditions, except that we made a normal time run and did not try to save model. The results for the "slow" and "fast" runs were identical.

A. VAN DER NEUT: The models used were flexible in bending and torsion and had negligible deformation in chordwise sense. For actual low aspect ratio wings this deformation will be important. Would you expect, Dr. Ashley, that your conclusion with the 2 degrees of freedom that available theory predicts flutter very well may be extended to the case in which the chordwise deflections have to be accounted for?

H. ASHLEY and J. R. MARTUCCELLI: No, as based on our observations, but other results suggest optimism on this point.

D. J. JOHNS: Regarding the subject of choice of modes, are the authors aware of any experimental evidence to suggest a coupling between the lifting surface modes of deformation (i.e. torsion and bending) and chordwise bending modes of single (or more) wing skin panels. I am in fact suggesting that there may be a coupling between "classical" wing flutter and panel flutter. Might not such a coupling explain the flutter observed in the NACA hot jet experiments reported in the paper?

H. ASHLEY and J. R. MARTUCCELLI: None.